A non-singular infra-red flow
from D=5 gauged supergravity

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Abstract

We discuss domain wall solutions of 5-dimensional supergravity corresponding to a cosine-superpotential, which is derived by a gauging of the two Abelian isometries of the scalar coset $SU(2,1)/U(2)$. We argue that this potential can be obtained from M-theory compactification in the presence of G-fluxes and an M5-brane instanton gas. If we decouple the volume scalar of the internal space, the superpotential allows for two extrema, which are either ultra-violet or infra-red attractive. Asymptotically we approach therefore either the boundary or the Killing horizon of an anti-deSitter space or flat spacetime for a vanishing cosmological constant. If the volume scalar does not decouple, we obtain a run-away potential corresponding to dilatonic domain walls, which always run towards a vanishing cosmological constant.

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1 Introduction

In the past years many aspects of domain wall (DW) solutions of 5-dimensional supergravity have been discussed and one of the most interesting application concerns a supergravity description of the infra-red (IR) physics of 4-dimensional field theories. In particular it is possible to obtain a supergravity description of the renormalization group (RG) flow [1, 2, 3] towards non-trivial IR fixed points, which can be conformal or non-conformal. Most progress has been made for non-conformal IR fixed points [4, 5], where the scalars flow towards a singularity in the superpotential. Many superpotentials coming from string theory compactifications have poles, which are IR attractive [6] and imply a singularity in the supergravity solution [7, 8]. In special cases the singularity indicates the appearance of a Coulomb branch [9] or can be resolved as discussed in [10]. On the other hand less is known about conformal IR fixed points. Especially for supersymmetric RG-flows the construction of the corresponding supergravity solutions prove difficult [11] and the only known example that we are aware of has been discussed in [2, 12]. Moreover, potentials with IR-attractive fixed points are essential for a string- or M-theory embedding of the Randall-Sundrum (RS) scenario [13]. Only if such potentials exist a thin-wall approximation is justified and we can approximate the scalars as constants given by their fixed-point values.

In this letter we discuss domain wall solutions, where the superpotential in the simplest case depend on a single scalar θ and has the form

$$W = 2 \left(a + b \cos \theta \right) . \tag{1}$$

Potentials of this type are generated naturally by an instanton/monopole condensation and have an old history in gauge theory in the discussion of confinement [14] and for domain walls [15]; for a recent discussion see also [16].

As we show in the next section, this superpotential can be obtained from a model where the scalar fields parameterize the coset $SU(2,1)/U(2)$ and a linear combination of both Abelian isometries is gauged. Depending on the constant parameters a and b it yields ultra-violet (UV) as well as IR attractive fixed points and the extrema of W give a (negative) cosmological constant yielding an anti deSitter spacetime. In the special case of $a = \pm b$, the cosmological constant vanishes at one extrema and we obtain flat spacetime.

In section three we derive explicit domain wall solutions interpolating between the (different) extrema of W. Depending the choice for a and b, these solutions are not only interesting from the RG-flow point of view, but provide also a realization of the RS scenario; the no-go statements [11] do not apply for this model.

Embedding this model into N=2, D=5 gauged supergravity [17], the scalars parameterizing the $SU(2,1)/U(2)$ coset build up the universal hypermultiplet. As we argue in the last section, from the M-theory perspective the potential can be understood by a superposition of non-trivial G-fluxes and an M5-brane instanton gas.

2 Abelian gauging of the $SU(2,1)/U(2)$ coset

In 5-d supergravity obtained by string or M-theory compactification, the scalars parameterize a coset space and the only potential consistent with supersymmetry comes from gauging of isometries of this coset. Since we are interested in flat domain walls, we can omit the gauge fields and the bosonic part of the action reads

$$S = \int d^5x \ \sqrt{-g} \left[\frac{1}{2} R - V(\Phi) - \frac{1}{2} g_{MN} \partial_\mu \Phi^M \partial^\mu \Phi^N \right] \tag{2}$$

where the scalars Φ^M are real and parameterize a space with the metric g_{MN}. Following general arguments [18], the potential can be expressed in terms of a superpotential W as

$$V = 6 \left(\frac{3}{4} g^{MN} \partial_M W \partial_N W - W^2 \right) . \tag{3}$$

Before we can discuss BPS domain wall solutions, we have to derive the superpotential. We consider the coset $SU(2,1)/U(2)$ and the metric can be derived from the Kähler potential (in [19] more general models will be discussed)

$$K = -\log(1 - |z_1|^2 - |z_2|^2) , \qquad |z_1|^2 + |z_2|^2 < 1 . \tag{4}$$

There is another commonly used parameterization of this coset, which makes the quaternionic nature of this manifold manifest and corresponds to the Kähler potential $K = -\log(\frac{S+\bar{S}}{2} - C\bar{C})$ [20], where the complex scalars S, C are known to enter the universal hypermultiplet of $N = 2$ supergravity. In the ungauged case both parameterizations are equivalent, but after gauging the resulting superpotentials differ significantly[2]; in the quaternionic formulation we could not find a superpotential with at least two extrema. This may be related to the fact, that it is non-trivial to introduce global quaternionic coordinates on a curved space, which would be necessary for the discussion of domain walls; but this issue deserves further clarification [19]. We do not want to go into details here about the relation of the two models and will instead come back to our parameterization given by the Kähler potential (4). So, we will treat this 4-d scalar manifold not as a quaternionic but as a special Kähler space and follow the formalism developed in the literatur [22]. A disadvantage is however, that it is not clear whether this procedure yields necessarily supersymmetric solution with four unbroken supercharges (because in the case at hand z_1 and z_2 are in the same multiplet). A consistency check is, that the scaling dimensions coming from the sugra scalars fit into known representations in the dual field theory. We come back to this point below.

The two phase rotations of z_1 and z_2 are two Abelian isometries corresponding to the Killing vectors

$$k_1 = z_1 \partial_{z_1} - \bar{z}_1 \partial_{\bar{z}_1} \qquad , \qquad k_2 = z_2 \partial_{z_2} - \bar{z}_2 \partial_{\bar{z}_2} . \tag{5}$$

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[2] A similar effect is also known for $N = 4$, $D = 4$ supergravity, where the $SO(4)$ and $SU(2) \times SU(2)$ formulation are duality equivalent in the ungauged case, but differ after gauging. The first case has an AdS vacuum, the other not, see [21]. Note, gauging does not preserve the duality symmetry.

It appears more convenient to introduce polar coordinates, as in [23], given by

$$z_1 = r \left(\cos \theta/2\right) e^{i(\psi + \varphi)/2} \qquad , \qquad z_2 = r \left(\sin \theta/2\right) e^{i(\psi - \varphi)/2} \qquad (6)$$

and the scalar field metric $ds^2 = \partial_{\bar{z}^i} \partial_{z^j} K \, d\bar{z}^i dz^j$ becomes

$$ds^2 = \frac{dr^2}{(1 - r^2)^2} + \frac{r^2}{4(1 - r^2)^2} \left(d\psi + \cos \theta \, d\varphi\right)^2 + \frac{r^2}{4(1 - r^2)} \left(d\theta^2 + \sin^2 \theta \, d\varphi^2\right) . \qquad (7)$$

The Kähler form is exact and can be written as

$$K_{uv} \, dq^u \wedge dq^v = d\hat{\omega} \, , \qquad \text{where} \quad \hat{\omega} = \frac{i}{2} \frac{r^2}{1 - r^2} \tau_3 \, , \quad \tau_3 = d\psi + \cos \theta \, d\varphi \qquad (8)$$

with $q^u = (r, \theta, \varphi, \psi)$. Next, we gauge a general linear combination of the two Killing vectors k_1, k_2 with some constants a, b

$$k = a \left(k_1 + k_2\right)/2 + b \left(k_1 - k_2\right)/2 = -4i \left(a \, \partial_\psi + b \, \partial_\varphi\right) \qquad (9)$$

and the scalar derivative becomes $D_\mu q^u = \partial_\mu q^u + k^u A_\mu$, where A_μ is the graviphoton (we do not consider vector multiplets). The superpotential is given by the Killing prepotential $W = P$ which is defined by the Kähler 2-from [22]

$$K_{vu} k^u = -\partial_v P \qquad (10)$$

and becomes

$$W = P = \frac{r^2}{2(1 - r^2)} \left(a + b \cos \theta\right) . \qquad (11)$$

In addition, the Killing spinor equations get corrected by $\sim W \Gamma_\mu \epsilon$ for the gravitino variation and $\sim i k^u \epsilon$ for the hyperino variation. There are numerous different conventions yielding different factors in these variations, but they can be fixed by the condition, that the vacuum is given by extrema of W and that the difference of the extrema on each side of the wall gives the energy stored by the wall. Moreover, solutions of the BPS equations solve also the equations of motion for our Lagrangian (2); we come back to these equations in the next section.

3 BPS domain wall solution

In supergravity one refers to domain walls as kink solution interpolating between different extrema of the potential. As an Ansatz for the metric which is adapted to the RG flow interpreation and preserves 4-d Poincare invariance one can use

$$ds^2 = \mu^2 \left(- dt^2 + d\vec{x}^2\right) + \frac{d\mu^2}{\left(\mu \widehat{W}(\mu)\right)^2} \, , \qquad (12)$$

where the fifth coordinate μ will be identified with an energy scale in the dual 4-d field theory. In these coordinates the UV region (= large length scale in supergravity) corresponds to $\mu \to \infty$, whereas the IR is approached for $\mu \to 0$. For our purpose we are only interested in the dependence of the scalar fields on the fifth coordinate or, in terms of the dual field theory, on the energy scale μ. Using this ansatz for the metric the first order equations that solve the equations of motion are [24]

$$W = \pm \widehat{W} \qquad , \qquad \beta^M \equiv \mu \frac{d}{d\mu} \Phi^M = -3 \, g^{MN} \partial_N \log W \ . \tag{13}$$

Using the projector $\Gamma^5 \epsilon = \pm \epsilon$ the first equation is equivalent to the gravitino and the second to the hyperino variation. Oviously, supersymmetric extrema of V occur at $\partial_M W = 0$ where also $\beta^M = 0$ holds. For $W|_{\partial W=0} \neq 0$ one has an AdS space with W being the cosmological constant while $W|_{\partial W=0} = 0$ corresponds to a flat space time. From the field theory point of view, β^M is a natural candidate for the β-functions of the couplings related to the supergravity scalar fields. For BPS solutions these functions determine the holographic RG flow [3, 24].

The nature of the fixed point is determined by the derivatives of the β-functions[3]

$$\partial_N \beta^M |_{\beta=0} = -3 \, g^{MK} \frac{\partial_N \partial_K W}{W} \Big|_{\beta=0} \ . \tag{14}$$

In the case that all scalars are in vector multiplets one finds $\partial_M \beta^N |_{\beta=0} = -2\delta_M^N$ and all fixed points are necessarily UV attractive [24, 11]. Consequently, these models cannot describe a smooth RG flow or give a smooth RS scenario, where each side of the wall has to be IR attractive. This situation is very typical for many potentials coming from string or M-theory compactification, which do not allow for "good" domain walls with at least two smoothly connected extrema. The situation in four dimensions is better [25] (for a review see [26]), but 5-d supergravity is more restrictive. In fact, generically one has no isolated extrema and instead a "run-away" potential giving rise to dilatonic domain walls. This behaviour is caused by the scalar field parameterizing the volume of the internal space. Whenever this scalar is dynamical it runs either to zero or infinite volume. There is no mechanism known to stabilize the volume at a finite value, at least not in a supersymmetric way. This happens also in our case, where the radial part allows only for a single extrema where the superpotential vanishes and the spacetime becomes flat. On the other hand the volume scalar can become non-dynamical. For example, quantum corrections in string or M-theory can provide a natural lower bound on the volume [27], which cut-off the radial flow, see [28]. Or, adopting the procedure discussed for 4-d vacua [29], we can consider an infinite volume limit (non-compact Calabi-Yau), which becomes equivalent to treat the r-coordinate as non-dynamical (i.e. constant). In doing this we can normalize $g_{\theta\theta} = 1$ and consider first the model for the superpotential

$$W = 2 \, (a + b \cos \theta) \ . \tag{15}$$

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[3]Here we assume that the fixed point is non singular i.e. the metric non-degenerate.

At the end we will also comment on the solution coming from a running radial coordinate. Coming back to our analysis from before, this superpotential has the properties we are looking for. It has two extrema where $\cos\theta = \pm 1$ with

$$\partial_\theta \beta^\theta = 3b \frac{b + a\cos\theta}{(a + b\cos\theta)^2} \quad , \quad \Delta = \partial_\theta \beta^\theta\big|_{\beta=0} = \frac{3b}{b \pm a} \tag{16}$$

where Δ corresponds to the scaling dimension of the corresponding field theory operator. Therefore, for $a > b > 0$ it describes a flow from the UV at $\cos\theta = -1$ towards the IR at $\cos\theta = 1$. If $b > a \geq 0$ we have two IR fixed points and similarly for $0 > a > b$ one finds on each side of the wall an UV fixed point. Finally for $a = b > 0$ it is a domain wall between an IR point at $\cos\theta = 1$ and flat spacetime at $\cos\theta = -1$. Thus, this simple model describes all known types of supersymmetric domain wall solutions. If both sides have the same type of fixed points, W has to change its sign and the solution cannot be interpreted as RG-flow, see also [30]. The pole in the β-functions indicates a first order phase transition.

As we mentioned earlier, in order to trust the embedding of our model into $N = 2$ supergravity, we have to ensure, that the scaling dimensions fit into short representations of superfields of the dual field theory, see e.g. [2], and there are some interesting cases. If we gauge e.g., only the ∂_φ isometry we obtain $a = 0$ and $\Delta = 3$; if we gauge only k_1 or k_2 we have $a = \pm b$ with $\Delta = \frac{3}{2}$ or if we turn off the ∂_φ gauging there are no running hyper scalars and we obtain a special case of the model discuss in [24] with $\Delta = 2$ for the vector scalars.

Next, let us construct the explicit domain wall solution. The coordinate system used before was adapted for the discussion of the RG flow, but in order to find an explicit solution we write the metric as

$$ds^2 = e^{2A(z)}\left(-dt^2 + d\vec{x}^2\right) + dz^2 . \tag{17}$$

For these coordinates the BPS equations become [7]

$$\partial_z A = W , \qquad \partial_z \theta = -3\, g^{\theta\theta} \partial_\theta W . \tag{18}$$

and inserting the superpotential (15) and $g_{\theta\theta} = 1$, we find as solution

$$e^{2A} = e^{4az}\left(\cosh 6bz\right)^{-2/3} , \qquad \cos\theta = -\tanh 6bz . \tag{19}$$

Approaching the two AdS vacua at $z = \pm\infty$, the warp factor becomes $e^{2A} \simeq e^{4(\pm a - b)|z|}$ and as mentioned before if $a > b > 0$ we have an UV fixed point at $z = +\infty$ and an IR fixed point on the other side. If $b > a > 0$ there are IR fixed point on both sides, which becomes Z_2 symmetric if $a = 0$. In this case, e^{2A} is exponentially decreasing on both sides yielding a localization of gravity on the wall and our model describes a Randall-Sundrum scenario [13]. Having this thick wall, one can also consider a thin-wall approximation ($b \to \infty$), where the scalar becomes constant and the spacetime

is everywhere AdS, up to the discontinuity at $z = 0^4$. Another interesting example is $a = b$, where we find

$$e^{-2A} = (1 + e^{-12az})^{2/3} \tag{20}$$

and the domain wall represents a flow from an IR fixed point at $z = -\infty$ to flat spacetime at $z = +\infty$.

Finally let us comment on the case of a running radius r, i.e. we consider the complete superpotential (11). In the θ-equation in (18), the radial part drops out and we obtain the same solution $\cos\theta = -\tanh 6bz$ and in addition we have the equation for the radius

$$\partial_z r = -3g^{rr}\partial_r W = -3r(a + b\cos\theta) = -3(a - b\tanh 6bz) \tag{21}$$

which is solved by $r^2 = e^{-6az}\cosh 6bz$ for $a > b$ (keeping in mind, that $0 \leq r < 1$). Therefore, $z = 0$ corresponds to the singular point $r = 1$, where the superpotential and the potential V have a pole and the warp factor in the metric has a zero: $e^{2A} \sim z^{1/6}$ for $z \simeq 0$. Towards larger values of z the warp factor increases and for $z = +\infty$ corresponding to $r = 0$ the potentials vanish and the spacetime becomes flat. Hence, taking the radial part into account, there are neither UV nor IR fixed points related to an AdS spacetime. Notice that using our definition from before, the β-function $\beta^r = -6(\frac{1}{r} - r)$ vanishes only at the singularity $r = 1$, which is IR attractive, as the cases discussed in [6].

4 Some comments about the M-theory embedding

Perhaps the easiest way to understand the solution comes from M-theory, where the four scalars of the universal hypermultiplet can be understood as follows. One scalar parameterizes the volume of the internal space, one axion comes from the dualization of the 5-d 4-form field strength, and two scalars are related to membranes wrapping $\Omega_{0,3}$ or $\Omega_{3,0}$ cycle. In our parameterization we gauged the Killing vector $k \sim a\,\partial_\psi + b\,\partial_\varphi$. The case $b = 0$ reproduces the results derived in [32, 28] and therefore corresponds to turning on 4-form fluxes in the M-theory compactification, which is equivalent to a non-trivial M5-brane background. On the other hand gauging the ∂_φ isometry gives a mass term for the θ axion. In fact for this deformation (setting $a = 0$) the supergravity potential can be written as (normalizing $g_{\theta\theta} = 3/4$ and rescaling $\theta \to \frac{2\theta}{\sqrt{3}}$): $V = -6b\cos\frac{4\theta}{\sqrt{3}}$ and we obtain a sine-Gordon model coupled to gravity. This form suggests, that it can be understood as coming from a non-trivial instanton background. M-theory compactified to 5 dimension may give a topological term $\int d^5x\,\theta\,dG$. If there are no sources present, the Bianchi identity implies $dG = 0$ and such a term vanishes. If on the other hand magnetic sources for G exist in 5 dimensions, then dG is non-vanishing, and this term is the 5-d analogue of the familiar universal axionic coupling $\int d^4x\,\psi\,tr(F \wedge \tilde{F})$ in 4 dimensions. An obvious candidate for magnetic sources of G are

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[4]Supersymmetric versions of the Randall-Sundrum geometry have also been discussed in refs. [31].

M5-branes. However in order to generate a cosine potential for θ such sources must be pointlike in 5 dimensions (like the instanton density $tr(F \wedge F)$ in 4 dimensions) and therefore correspond to Euclidean M5-branes wrapping the whole CY manifold, in contrast to the 5-branes associated with the gauging of the ∂_ψ isometry, which wrap a holomorphic 2-cycle and become 3-branes upon compactification. As in 4 dimensions, see [33], we expect that the sum over an M5-brane instanton gas will reproduce the cosine potential. Clearly, this interpretation deserves a more detailed investigation.

Acknowledgements

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